



The parabolic method for the allotment of seats in the European Parliament among Member States of the European Union

Victoriano Ramírez González *

Theme: This analysis proposes a method for the objective and fair distribution of the seats in the European Parliament among all European Union Member States.*

Summary: After a number of failed attempts, the exact formula for the allotment of seats to each member state within the European Parliament continues to be an unresolved issue. The Intergovernmental Conference of 2004 agreed upon the maximum and minimum thresholds that should be respected (96 seats for the largest countries and six for the smallest ones) as well as on a guiding principle, “digressive proportionality” for the distribution of the remainder of seats. However, the agreement, which took shape in article I-20 of the European Constitution, does not specify what formula should be used for the allotment of the rest of these seats. In the past, the solution to this problem has entailed political negotiations, with each State wielding its reasons, its continental prestige, its concessions in other areas, and its capacity for applying pressure and persuasion in order to maximize its quota of seats. Such a system rewards the negotiating efficiency and the relative power of the Member States, and it will tend to produce precarious results from the standpoint of stability, and even of the legitimacy of the system. The alternative is to search for a mathematical formula that translates the principle of digressive proportionality to the scenario of reality in the soundest way possible. This would ensure a discretionary nature to the allotment of seats while also —by adjusting faithfully to the principle of digressive proportionality— enhancing the legitimacy and stability of the system. This analysis presents and discusses to what point the parabolic method could provide a formula for seat allotment satisfying such requisites. With this formula, the principle of digressive proportionality is respected, and the seats in the European Parliament are distributed in a fair way. The result is that the largest States have more seats than the smallest ones, but fewer than those that would correspond to them in proportion to their population; and vice versa for the smallest states. Thus, it is a very interesting allotment method, accounting for those political and technical criteria that should govern the distribution of seats in the European Parliament.

* *Professor of Universidad de Granada*

* The author would like to thank the Junta de Andalucía for support through its group FQM-191 and the Project of Excellence FQM-01969 which made it possible to cover the costs of research in proportional representation and social choice; as well as the Department of Applied Mathematics of the University of Granada for its research incentives.

Analysis:

Introduction

This study shows the results of applying the parabolic method¹ for the allotment of seats in the European Parliament among the 27 Member States of the European Union, taking into account the restrictions established in article I-20 of the project of the European Constitution. It also presents a simulation for the inclusion of Croatia as the 28th member of the EU. The parabolic method for the distribution of the seats of the European Parliament can be applied to the current composition of the EU or, just the same, if there are variations in the number of states, their populations, or the size of Parliament, or if the limitations established in the projected European Constitution lead to other possible configurations. Hence, it is a method which, because of its flexibility, would permit the achievement of a stable framework that would not have to be revised with each successive addition to the number of EU members or changes in their populations.

Assignment of the seats of the European Parliament. Limitations established in the project of the European Constitution.

So far, the allotment of seats within the European Parliament has not followed criteria of *proportionality*, as the small countries have been over-represented with respect to the large ones. For instance, Luxembourg has at present six seats (the same number as in previous legislatures), when its *exact quota*² is less than one; and Malta, which is the smallest Member State, has five representatives at present. Meanwhile, the countries with a greater number of population have always been under-represented: Germany now has 99 representatives, though its quota on the basis of population is over 125.

The allotment of the seats of the European Parliament (EP) among the States of the Union has always been settled by negotiation. Evidently, negotiating is not a mechanism that can be adopted indefinitely. Indeed, the project of the European Constitution indicates that the European Council should put forth a decision establishing the makeup of the European Parliament. It seems logical that such a decision would contain an unambiguous formula to determine the distribution of seats among the Member States. This formula should be valid for assigning seats to the current members, and must also be valid if the number of EU states varies, or their populations fluctuate. Moreover, the Constitutional project sets forth requisites that are in agreement with tradition: for example, the Parliament may not have a size over 750 seats, and no country of the Union may receive fewer than six seats or more than 96. A further requisite is that a rule of *digressive proportionality* should be applied when allotting the seats.

The problem with this is that the project of the European Constitution does not explain what is to be understood by digressive proportionality, which makes it almost obligatory to open discussion as to what type of formula or formulas might best translate this principle into reality. For reasons stated already below, the so-called "parabolic method" offers an appropriate option. To illustrate the strong points of this method, it has been applied on the basis of the limitations established in the project of the European Constitution, regarding both the total size of the European Parliament (750 members), and in its upper

¹ The parabola method was presented for the first time at the "Workshop on Institutions and Voting Rules in the EU", celebrated in December 2004, in Seville (Spain). One description from the standpoint of political science, and another version with emphasis on mathematical contents (including several alternative methods) have been published in *Revista Española de Ciencia Política* and in a book on mathematics and democracy published by Springer Verlag (2006). At present, the parabola method is being revised to update the data on population, and to improve the presentation of tables and graphs, and the mathematical foundation. The tables appearing in this study contain the updated information.

² "Exact quota" means the seats corresponding to a country in proportion to its population. Thus, while Luxembourg has just 0.09% of the population of the EU-27 (459,000 inhabitants out of 492 million), in a European Parliament of 750 seats, it would obtain 0.62 seats.

and lower restrictions (96 seats for the largest countries and six for the smallest), limitations that we assume will still remain in the near future.

The bases of the parabolic method

The *quota* (also known as *exact quota*) of each country is the fraction of seats that would correspond to that country in strict proportion to its population. In proportional distributions, a method is applied to round off the quotas to the nearest integers.

Yet the allotment of the seats of the EP is not proportional, requiring *digressive proportionality* and certain additional limits (minimum of six seats and maximum of 96 for any country).

The parabolic method is based on a function of the type $A(x) = a + bx + cx^2$, whose graph is a parabola of the second degree, in order to transform the exact quota of a given country into another value called the *adjusted quota*. The objective of this transformation is to achieve digressive proportionality while remaining within the limits of 6 to 96. Afterwards, the adjusted quotas are rounded off to natural numbers using the Webster or the Sainte-Laguë method (of proportional allotment).

The use of a parabolic-type function, which has three free coefficients (a , b , c), allows us to get:

- i. An adjusted quota for the smallest countries that coincides with the lower limitation (at present it would be 6, corresponding to Malta).
- ii. An adjusted quota for the countries with a greater number of population that coincides with the upper limit (at present it would be 96, corresponding to Germany).
- iii. A total sum of adjusted quotas that coincides with the size of the EP (for the simulations we will use 750, as the maximum permitted).

In the theory of polynomial interpolation, it is known to exist a function of parabolic type that fulfils these three conditions, as long as the number of states of the EU is greater than two. Moreover, for the allotment of the seats among the 27 current member states, and in the face of the eventual incorporation of Croatia, the function obtained imposing these three conditions would allow us to obtain the allotment under the parabolic method.

Notwithstanding, we need to consider all the situations that may arise. In other words, there are occasions for which the calculation of the function of adjusting quotas is carried out imposing only part of the above conditions.

From time to time it will be necessary to substitute one of the two first conditions in order to guarantee digressive proportionality. Accordingly, to give a few examples, if several more countries join the EU, Turkey among them, the exact quota of Germany would be below the value of 96, yet a readjustment of quotas in which Germany ends up with 96 seats would not provide for digressive proportionality. This situation will be possible in the near future. The opposite case scenario would be for the smallest countries to abandon the EU, which would cause the smallest remaining country to have an exact quota over 6, and if this were lowered to 6, it would not obey the notion of digressive proportionality. The latter situation, granted, appears unlikely as things now stand.

Basically, to ensure that the adjustment of quotas is digressively proportional, we must assure that the quota adjustment function $A(x)$ verifies the conditions:

1. *Growth.*

$$x < y \quad \text{implies that} \quad A(x) < A(y)$$

That is, if the exact quotas of two States are “ x ” and “ y ”, verifying $x < y$, then the corresponding adjusted quotas should verify the same relation. One particular state with a greater population than another cannot be given fewer representatives [2].

From a mathematical point of view, this means that the derivative of the quota adjustment function must be positive: $A'(x) \geq 0$, $\forall x \in [m, M]$, where m and M are the quotas of the least populated and the most populated countries, respectively.

If the calculation of $A(x)$ gives a negative derivative at the endpoint, we are bound to increase the value 6 of condition i. to a greater value (which will be the value for which the derivative at the endpoint will be zero). If the derivative of $A(x)$ were negative at the initial point, the function of adjustment would be calculated in a rectilinear fashion, imposing only conditions i. and iii.

2. *Concavity.*

Quotas stand for exact proportionality, that is, the adjustment function $A(x) = x$ (whose graph is the bisectrix of the first quadrant, a line with slope one), corresponds to exact proportionality, whereas the total absence of proportionality is the assignment of an equal number of representatives to all the States, corresponding to the function $A(x) = c$ (constant function, therefore with slope zero).

Thus, in the second place, it is necessary to interpret the concept of *increasing proportionality*, which implies using a function $A(x)$ whose slope is decreasing or, at least, does not increase (concave function). Therefore we must ensure:

$$A''(x) \leq 0, \quad \text{para todo } x \in [m, M].$$

If this condition is not verified, we are bound to assign fewer than 96 seats to the largest country. In such a case, the adjustment function is rectilinear, verifying conditions i. and iii. Hence, if the number of EU countries continues to grow in future, the graph of the quota adjustment function of the parabolic method will be a straight line (its calculation acquiring maximum simplicity).

By using the parabolic method, it is intended that the smallest country get no more seats than the minimum established in the project of the Constitution; nor should the largest country get fewer than the maximum stipulated (these being the most logical references to use at present). Sometimes it is impossible to enforce these limitations, be it with the parabolic method or any other one; in such a case, the parabolic method adjusts one of the thresholds and approximates the other to a maximum. Obviously, the allotments will always comply with constitutional requisites, since the projected text states, for example, that no state can receive more than 96 seats, yet it does not force the one with greater number of population to receive exactly 96 —fewer may be assigned.

The calculation of adjusted quotas using the parabolic method always gives a unique solution. For rounding out the adjusted quotas, the Webster method is proposed (or else

the Sainte Laguë method) in view of its qualities of consistency, monotony and impartiality. This method approximates the adjusted quotas to the nearest integer. As in any process of proportional distribution of course, in this phase multiple solutions may be produced (that is, ties); although in practice they are very improbable.

Allotment with the parabolic method of 750 EP seats among the 27 current Member States.

The maximum limit for the size of the EP is 750 seats. The EU Parliament of the 25 Member States, before 2007, had 732 seats and foresee the entry of Bulgaria and Rumania in 2007, and so the current size is substantially greater than 750. Yet it is most likely that, in future, the maximum limit of 750 Parliamentary seats be respected; hence, it is the figure we use for the simulations presented below.

Table 1: Parabolic allotment of 750 seats among the 27 Member States of the EU

Country	Inhabitants	Quota	Adjust.Q.	Differ.	Parabolic	Present
Germany	82,438,000	125.44	96.00	-23.47 %	96	99
France	62,886,200	95.69	78.78	-17.67 %	79	78
United Kingdom	60,421,900	91.94	76.43	-16.87 %	76	78
Italy	58,751,700	89.40	74.81	-16.33 %	75	78
Spain	43,758,300	66.58	59.38	-10.82 %	59	54
Poland	38,157,100	58.06	53.21	- 8.35 %	53	54
Rumania	21,610,200	32.88	33.74	2.60 %	34	-
Netherlands	16,334,200	24.85	27.13	9.14 %	27	27
Greece	11,125,200	16.93	20.41	20.58 %	20	24
Portugal	10,569,600	16.08	19.69	22.40 %	20	24
Belgium	10,511,400	15.99	19.61	22.60 %	20	24
Czech Rep.	10,251,100	15.60	19.27	23.52 %	19	24
Hungary	10,076,600	15.33	19.04	24.16 %	19	24
Sweden	9,047,800	13.77	17.68	28.43 %	18	19
Austria	8,265,900	12.58	16.65	32.35 %	17	18
Bulgaria	7,718,800	11.75	15.92	35.54 %	16	-
Denmark	5,427,500	8.26	12.85	55.62 %	13	14
Slovakia	5,389,200	8.20	12.80	56.09 %	13	14
Finland	5,255,600	8.00	12.62	57.81 %	13	14
Ireland	4,209,000	6.40	11.21	74.97 %	11	13
Lithuania	3,403,300	5.18	10.11	95.26 %	10	13
Latvia	2,294,600	3.49	8.60	146.28 %	9	9
Slovenia	2,003,400	3.05	8.20	169.00 %	8	7
Estonia	1,344,700	2.05	7.30	256.57 %	7	6
Cyprus	766,400	1.17	6.50	457.35 %	6	6
Luxembourg	459,500	0.70	6.08	769.02 %	6	6
Malta	404,300	0.62	6.00	875.30 %	6	5
Total EU-27	492,881,500	750.00	750.00	-----	750	732

Observations:

The column "Inhabitants" contains the populations of the 27 Member States for the year 2006. Source: Council Resolution on January 1st, 2007 modifying the Internal Regulation 2007/4/CE, Euroatom, EU Official Diary 4.1.2007 L1/Pp. 9-10

The column "Quota" contains the exact quotas or proportions.

The column "Adjust.Q." contains the parabolic adjustment of the quotas (digressively proportional) subjected to the minimum of 6 and maximum of 96, which we shall call *adjusted quotas*. The quota adjustment function applied was:

$A(x) = 5,4609 + 0,9097x - 0,001497x^2$, and the value was obtained imposing the three conditions: i., ii. and iii. It verifies conditions 1. and 2. of digressive proportionality.

The column "Differ." shows the percentage of loss or gain by each country when the exact quota is transformed to the adjusted quota. *It is seen that the larger the country, the greater the loss in quota, and the smaller the country, the greater its gain in quota.* Below, in the column

"Parabolic", is the allotment obtained through the parabolic method, which consists of applying Webster's method (or Sainte-Laguë's) to the adjusted quotas of the column "Adjust.Q."

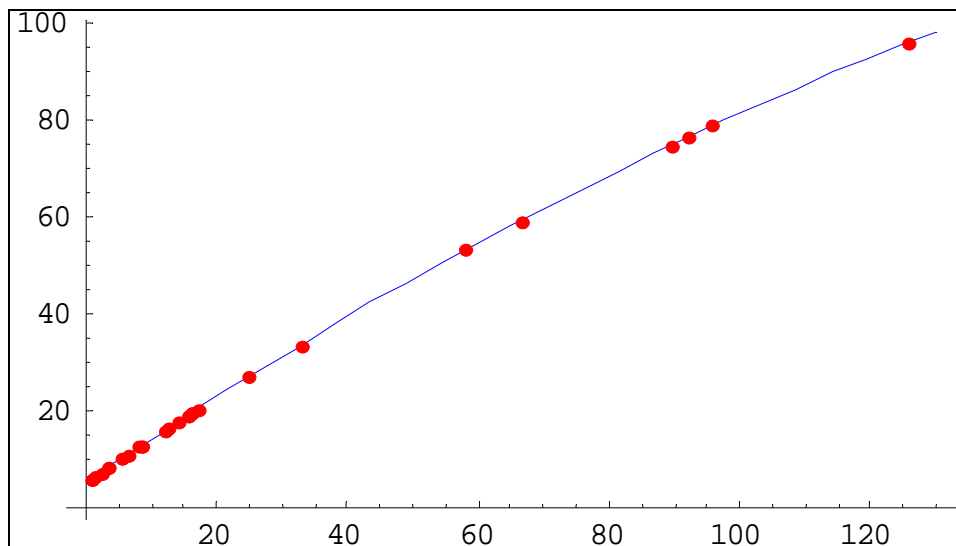
The final column ("present") shows the real allotment at present.

Observations relative to the comparison between parabolic allotment and real allotment.

Upon comparison of the present allotment with the theoretical parabolic allotment for one particular country, one should not jump to the conclusion that a given country get more benefits through one method or the other merely on the basis of a greater assignment of seats. Rather, two conditioning factors must be taken into account. Firstly, the present allotment is the result of negotiation years ago; and secondly, different data have been used:

- Populations have not increased to the same extent in all the Member States over recent years. Spain, for instance, is one of the countries with the greatest increase, whereas Germany has seen a substantial decrease in population.
- The minimum and maximum values have changed, from 5 and 99, to 6 and 96.
- The size of the Parliament has varied, from 732 to 750.
- Two new member States, Bulgaria and Rumania, forming part of the European Union as of 2007, receive a total of 50 seats between the two.

The graph of the allotment using the parabolic method for 27 States is:



The horizontal axis represents the countries' exact quotas, and the vertical axis shows the seats assigned. The blue curve is the quota adjustment parabola, which begins at (0.62-6) and ends at the point (125.44-96), that is, it adjusts (rounding off) the quota of Malta to 6 and that of Germany to 96. The red dots represent the allotments using the parabolic method.

Allotment using the parabolic method and considering the incorporation of Croatia: EU-28

In this case we shall assume that the European Union is expanded to include 28 Member States with the entry of Croatia (which will probably be the next country to join). The simulation is performed again with the present populations, a Parliamentary size of 750 seats, and the limits 6 and 96. The results obtained are shown in the last column of Table 2.

Table 2. Allotment using the parabolic method and incorporating Croatia

Country	Inhabitants	Quota	Adjust.Q.	% Differ.	Parabolic
Germany	82,438,000	124.32	96.00	-22.78	96
France	62,886,200	94.84	77.97	-17.78	78
U. Kingdom	60,421,900	91.12	75.55	-17.09	76
Italy	58,751,700	88.60	73.89	-16.60	74
Spain	43,758,300	65.99	58.27	-11.70	58
Poland	38,157,100	57.54	52.11	-9.45	52
Rumania	21,610,200	32.59	32.88	0.90	33
Netherlands	16,334,200	24.63	26.43	7.30	26
Greece	11,125,200	16.78	19.91	18.65	20
Portugal	10,569,600	15.94	19.20	20.46	19
Belgium	10,511,400	15.85	19.13	20.66	19
Czech Rep.	10,251,100	15.46	18.80	21.59	19
Hungary	10,076,600	15.20	18.57	22.23	19
Sweden	9,047,800	13.64	17.26	26.51	17
Austria	8,265,900	12.47	16.26	30.44	16
Bulgaria	7,718,800	11.64	15.56	33.65	16
Denmark	5,427,500	8.18	12.60	53.89	13
Slovakia	5,389,200	8.13	12.55	54.37	13
Finland	5,255,600	7.93	12.37	56.11	12
Croatia	4,442,800	6.70	11.31	68.87	11
Ireland	4,209,000	6.35	11.01	73.44	11
Lithuania	3,403,300	5.13	9.95	93.96	10
Latvia	2,294,600	3.46	8.50	145.60	9
Slovenia	2,003,400	3.02	8.12	168.60	8
Estonia	1,344,700	2.03	7.25	257.30	7
Cyprus	766,400	1.16	6.48	460.68	6
Luxembourg	459,500	0.69	6.07	776.42	6
Malta	404,300	0.61	6.00	884.07	6
Total EU-28	497,324,300	750.00	750.00	-----	750

Croatia receives 11 representatives, at the expense of 11 States that lose representatives in relation to the allotment appearing in Table 1. It is important to note, once again, that the amounts shown in the column "Difference" increase.

In this case, the quota adjustment function is: $A(x) = 5,4630 + 0,8816x - 0,001233x^2$, which verifies all the requisites of digressive proportionality and furthermore transforms Malta's quota to 6 and Germany's to 96.

In both simulations, the six countries with the greatest populations are the only ones to lose quota (the percentual loss being greater when the population is greater). The remaining 22 countries gain quota with the adjustment, and, percent-wise, the smaller the country, the more it gains in quota as a result of the adjustment.

Conclusion

This analysis has described a method (the “parabolic method”) with which the allotment of seats in the European Parliament may be determined in such a way that the requisites and principles established in article I-20 of the project of the European Constitution are fulfilled, respecting both the criterion of digressive proportionality and the maximum and minimum numbers of seats to be assigned to each State. It is a method that is not only fair but also easily adapted to any possible configuration in terms of the number of seats or member states of the European Union. For this reason, in addition to reducing the complexity of negotiations among Member States, it could prove useful in conferring the institutional system of the EU enhanced stability and legitimacy.

Victoriano Ramírez González
Professor of Universidad de Granada

References

Balinski, M. L., Young H. P., *Fair Representation: Meeting the Ideal of One Man One Vote*. New Haven, CT, 1982.

Bovens, L., Welfare, Voting and the Constitution of a Federal Assembly, <http://www.uni-konstanz.de/ppm/EU.pdf>

Ramírez V., *Some Guidelines for an Electoral European System*, Workshop on Institutions and Voting Rules in the EC. Seville (Spain), 2004.

Ramírez V., Palomares, A. Márquez, M. L., Un método para distribuir los escaños del Parlamento Europeo entre los Estados miembros de la UE, *Revista Española de Ciencia Política*, nº 14, pp. 71-85, 2006.

Ramírez V., Palomares, A. Márquez, M. L., Degressively proportional methods for the allotment of the European Parliament seats amongst the EU member States. *Mathematics and Democracy*, pp. 205-220. Berlin: Springer Verlag (2006).

Tratado por el que se establece una Constitución para Europa, *Ministerio de Asuntos Exteriores y de Cooperación, Ministerio del Interior y Ministerio de la Presidencia*, Dep. Legal M-53614/2004.

The program: MERPE (<http://www.ugr.es/local/vramirez>)